XXVIII

THE GENESIS OF DOUBLE STARS

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In ordinary speech a system of any sort is said to be stable when it cannot be upset easily, but the meaning attached to the word is usually somewhat vague. It is hardly surprising that this should be the case, when it is only within the last thirty years, and principally through the investigations of M. Poincaré, that the conception of stability has, even for physicists, assumed a definiteness and clearness in which it was previously lacking. The laws which govern stability hold good in regions of the greatest diversity; they apply to the motion of planets round the sun, to the internal arrangement of those minute corpuscles of which each chemical atom is constructed, and to the forms of celestial bodies. In the present essay I shall attempt to consider the laws of stability as relating to the last case, and shall discuss the succession of shapes which may be assumed by celestial bodies in the course of their evolution. I believe further that homologous conceptions are applicable in the consideration of the transmutations of the various forms of animal and of vegetable life and in other regions of thought. Even if some of my readers should think that what I shall say on this head is fanciful, yet at least the exposition will serve to illustrate the meaning to be attached to the laws of stability in the physical universe.

I propose, therefore, to begin this essay by a sketch of the principles of stability as they are now formulated by physicists.

I.

If a slight impulse be imparted to a system in equilibrium one of two consequences must ensue; either small oscillations of the system will be started, or the disturbance will increase without limit and the arrangement of the system will be completely changed. Thus a stick may be in equilibrium either when it hangs from a peg or when it is balanced on its point. If in the first case the stick is touched it will swing to and fro, but in the second case it will topple over. The first position is a stable one, the second is unstable. But this case is too simple to illustrate all that is implied by stability, and we must consider cases of stable and of unstable motion. Imagine a satellite and its planet, and consider each of them to be of indefinitely small size, in fact particles; then the satellite revolves round its planet in an ellipse. A small disturbance imparted to the satellite will only change the ellipse to a small amount, and so the motion is said to be stable. If, on the other hand, the disturbance were to make the satellite depart from its initial elliptic orbit in ever widening circuits, the motion would be unstable. This case affords an example of stable motion, but I have adduced it principally with the object of illustrating another point not immediately connected with stability, but important to a proper comprehension of the theory of stability.

The motion of a satellite about its planet is one of revolution or rotation. When the satellite moves in an ellipse of any given degree of eccentricity, there is a certain amount of rotation in the system, technically called rotational momentum, and it is always the same at every part of the orbit[1].

Now if we consider all the possible elliptic orbits of a satellite about its planet which have the same amount of "rotational momentum," we find that the major axis of the ellipse described will be different according to the amount of flattening (or the eccentricity) of the ellipse described. Fig. 1 illustrates for a given planet and satellite all such orbits with constant rotational momentum, and with all the major axes in the same direction. It will be observed that there is a continuous transformation from one orbit to the next, and that the whole forms a consecutive group, called by mathematicians "a family" of orbits. In this case the rotational momentum is constant and the position of any orbit in the family is determined by the length of the major axis of the ellipse; the classification is according to the major axis, but it might have been made according to anything else which would cause the orbit to be exactly determinate.

I shall come later to the classification of all possible forms of ideal liquid stars, which have the same amount of rotational momentum, and the classification will then be made according to their densities, but the idea of orderly arrangement in a "family" is just the same.

We thus arrive at the conception of a definite type of motion, with a constant amount of rotational momentum, and a classification of all members of the family, formed by all possible motions of that type, according to the value of some measurable quantity (this will

[1] Moment of momentum or rotational momentum is measured by the momentum of the satellite multiplied by the perpendicular from the planet on to the direction of the path of the satellite at any instant.

hereafter be density) which determines the motion exactly. In the particular case of the elliptic motion used for illustration the motion was stable, but other cases of motion might be adduced in which the motion would be unstable, and it would be found that classification in a family and specification by some measurable quantity would be equally applicable.

A complex mechanical system may be capable of motion in several distinct modes or types, and the motions corresponding to each such type may be arranged as before in families. For the sake of simplicity I will suppose that only two types are possible, so that there will

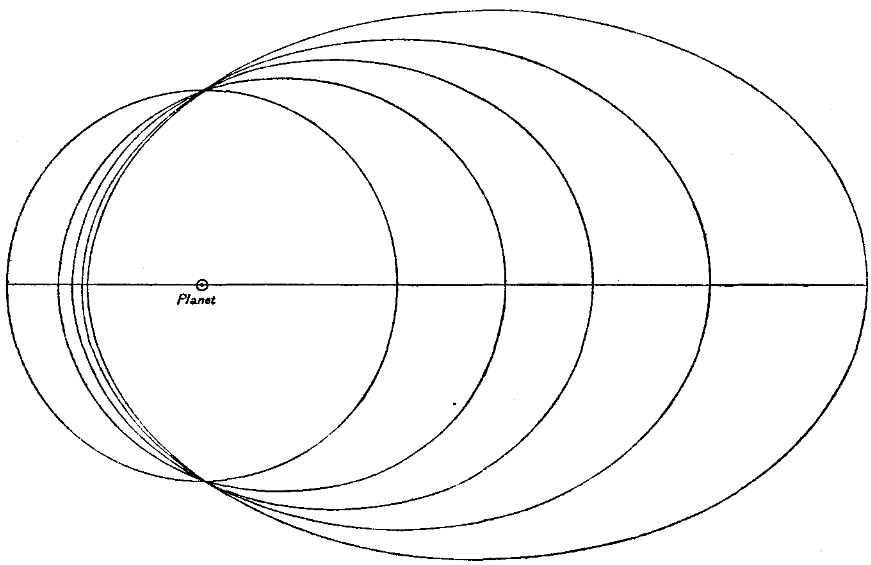

Fig. 1.

A "family" of elliptic orbits with constant rotational momentum.

only be two families; and the rotational momentum is to be constant. The two types of motion will have certain features in common which we denote in a sort of shorthand by the letter A. Similarly the two types may be described as $A + a$ and $A + b$, so that a and b denote the specific differences which discriminate the families from one another. Now following in imagination the family of the type $A + a$, let us begin with the case where the specific difference a is well marked. As we cast our eyes along the series forming the family, we find the difference a becoming less conspicuous. It gradually dwindles until it disappears; beyond this point it either becomes reversed, or else the type has ceased to be a possible one. In our shorthand we have started with $A+a$, and have watched the characteristic a dwindling to zero. When it vanishes we have reached a type which may be specified as A ; beyond this point the type would be $A-a$ or would be impossible.

Following the $A+b$ type in the same way, b is at first well marked, it dwindles to zero, and finally may become negative. Hence in shorthand this second family may be described as $A+b, \ldots A, \ldots A-b$.

In each family there is one single member which is indistinguishable from a member of the other family; it is called by Poincaré a form of bifurcation. It is this conception of a form of bifurcation which forms the important consideration in problems dealing with the forms of liquid or gaseous bodies in rotation.

But to return to the general question,—thus far the stability of these families has not been considered, and it is the stability which renders this way of looking at the matter so valuable. It may be proved that if before the point of bifurcation the type $A+a$ was stable, then $A+b$ must have been unstable. Further as a and b each diminish $A+a$ becomes less pronouncedly stable, and $A+b$ less unstable. On reaching the point of bifurcation $A+a$ has just ceased to be stable, or what amounts to the same thing is just becoming unstable, and the converse is true of the $A+b$ family. After passing the point of bifurcation $A+a$ has become definitely unstable and $A+b$ has become stable. Hence the point of bifurcation is also a point of "exchange of stabilities between the two types[1]."

In nature it is of course only the stable types of motion which can persist for more than a short time. Thus the task of the physical evolutionist is to determine the forms of bifurcation, at which he must, as it were, change carriages in the evolutionary journey so as always to follow the stable route. He must besides be able to indicate some natural process which shall correspond in effect to the ideal arrangement of the several types of motion in families with gradually changing specific differences. Although, as we shall see hereafter, it may frequently or even generally be impossible to specify with exactness the forms of bifurcation in the process of evolution, yet the conception is one of fundamental importance.

The ideas involved in this sketch are no doubt somewhat recondite, but I hope to render them clearer to the non-mathematical reader by

[1] In order not to complicate unnecessarily this explanation of a general principle I have not stated fully all the cases that may occur. Thus: firstly, after bifurcation $A+a$ may be an impossible type and $A+a$ will then stop at this point; or secondly, $A+b$ may have been an impossible type before bifurcation, and will only begin to be a real one after it; or thirdly, both $A+a$ and $A+b$ may be impossible after the point of bifurcation, in which case they coalesce and disappear. This last case shows that types arise and disappear in pairs, and that on appearance or before disappearance one must be stable and the other unstable.

homologous considerations in other fields of thought[1], and I shall pass on thence to illustrations which will teach us something of the evolution of stellar systems.

States or governments are organised schemes of action amongst groups of men, and they belong to various types to which generic names, such as autocracy, aristocracy or democracy, are somewhat loosely applied. A definite type of government corresponds to one of our types of motion, and while retaining its type it undergoes a slow change as the civilisation and character of the people change, and as the relationship of the nation to other nations changes. In the language used before, the government belongs to a family, and as time advances we proceed through the successive members of the family. A government possesses a certain degree of stability—hardly measurable in numbers however—to resist disintegrating influences such as may arise from wars, famines, and internal dissensions. This stability gradually rises to a maximum and gradually declines. The degree of stability at any epoch will depend on the fitness of some leading feature of the government to suit the slowly altering circumstances, and that feature corresponds to the characteristic denoted by a in the physical problem. A time at length arrives when the stability vanishes, and the slightest shock will overturn the government. At this stage we have reached the crisis of a point of bifurcation, and there will then be some circumstance, apparently quite insignificant and almost unnoticed, which is such as to prevent the occurrence of anarchy. This circumstance or condition is what we typified as b. Insignificant although it may seem, it has started the government on a new career of stability by imparting to it a new type. It grows in importance, the form of government becomes obviously different, and its stability increases. Then in its turn this newly acquired stability declines, and we pass on to a new crisis or revolution. There is thus a series of "points of bifurcation" in history at which the continuity of political history is maintained by means of changes in the type of government. These ideas seem, to me at least, to give a true account of the history of states, and I contend that it is no mere fanciful analogy but a true homology, when in both realms of thought—the physical and the political—we perceive the existence of forms of bifurcation and of exchanges of stability.

[1] I considered this subject in my Presidential address to the British Association in 1905, *Report of the 75th Meeting of the British Assoc.* (*S. Africa*, 1905), London, 1906, p. 3. Some reviewers treated my speculations as fanciful, but as I believe that this was due generally to misapprehension, and as I hold that homologous considerations as to stability and instability are really applicable to evolution of all sorts, I have thought it well to return to the subject in the present paper.

Further than this, I would ask whether the same train of ideas does not also apply to the evolution of animals? A species is well adapted to its environment when the individual can withstand the shocks of famine or the attacks and competition of other animals; it then possesses a high degree of stability. Most of the casual variations of individuals are indifferent, for they do not tell much either for or against success in life; they are small oscillations which leave the type unchanged. As circumstances change, the stability of the species may gradually dwindle through the insufficiency of some definite quality, on which in earlier times no such insistent demands were made. The individual animals will then tend to fail in the struggle for life, the numbers will dwindle and extinction may ensue. But it may be that some new variation, at first of insignificant importance, may just serve to turn the scale. A new type may be formed in which the variation in question is preserved and augmented; its stability may increase and in time a new species may be produced.

At the risk of condemnation as a wanderer beyond my province into the region of biological evolution, I would say that this view accords with what I understand to be the views of some naturalists, who recognise the existence of critical periods in biological history at which extinction occurs or which form the starting-point for the formation of new species. Ought we not then to expect that long periods will elapse during which a type of animal will remain almost constant, followed by other periods, enormously long no doubt as measured in the life of man, of acute struggle for existence when the type will change more rapidly? This at least is the view suggested by the theory of stability in the physical universe[1].

And now I propose to apply these ideas of stability to the theory of stellar evolution, and finally to illustrate them by certain recent observations of a very remarkable character.

Stars and planets are formed of materials which yield to the enormous forces called into play by gravity and rotation. This is obviously true if they are gaseous or fluid, and even solid matter becomes plastic under sufficiently great stresses. Nothing approaching a complete study of the equilibrium of a heterogeneous star has yet been found possible, and we are driven to consider only bodies of simpler construction. I shall begin therefore by explaining what is known about the shapes which may be assumed by a mass of incompressible liquid of uniform density under the influences of gravity and of rotation. Such a liquid mass may be regarded as

[1] I make no claim to extensive reading on this subject, but refer the reader for example to a paper by Professor A. A. W. Hubrecht on "De Vries's Theory of Mutations," *Popular Science Monthly*, July 1904, especially to p. 213.

an ideal star, which resembles a real star in the fact that it is formed of gravitating and rotating matter, and because its shape results from the forces to which it is subject. It is unlike a star in that it possesses the attributes of incompressibility and of uniform density. The difference between the real and the ideal is doubtless great, yet the similarity is great enough to allow us to extend many of the conclusions as to ideal liquid stars to the conditions which must hold good in reality. Thus with the object of obtaining some insight into actuality, it is justifiable to discuss an avowedly ideal problem at some length.

The attraction of gravity alone tends to make a mass of liquid assume the shape of a sphere, and the effects of rotation, summarised under the name of centrifugal force, are such that the liquid seeks to spread itself outwards from the axis of rotation. It is a singular fact that it is unnecessary to take any account of the size of the mass of liquid under consideration, because the shape assumed is exactly the same whether the mass be small or large, and this renders the statement of results much easier than would otherwise be the case.

A mass of liquid at rest will obviously assume the shape of a sphere, under the influence of gravitation, and it is a stable form, because any oscillation of the liquid which might be started would gradually die away under the influence of friction, however small. If now we impart to the whole mass of liquid a small speed of rotation about some axis, which may be called the polar axis, in such a way that there are no internal currents and so that it spins in the same way as if it were solid, the shape will become slightly flattened like an orange. Although the earth and the other planets are not homogeneous they behave in the same way, and are flattened at the poles and protuberant at the equator. This shape may therefore conveniently be described as planetary.

If the planetary body be slightly deformed the forces of restitution are slightly less than they were for the sphere; the shape is stable but somewhat less so than the sphere. We have then a planetary spheroid, rotating slowly, slightly flattened at the poles, with a high degree of stability, and possessing a certain amount of rotational momentum. Let us suppose this ideal liquid star to be somewhere in stellar space far removed from all other bodies; then it is subject to no external forces, and any change which ensues must come from inside. Now the amount of rotational momentum existing in a system in motion can neither be created nor destroyed by any internal causes, and therefore, whatever happens, the amount of rotational momentum possessed by the star must remain absolutely constant.

A real star radiates heat, and as it cools it shrinks. Let us suppose then that our ideal star also radiates and shrinks, but let the process proceed so slowly that any internal currents generated in the liquid by the cooling are annulled so quickly by fluid friction as to be insignificant; further let the liquid always remain at any instant incompressible and homogeneous. All that we are concerned with is that, as time passes, the liquid star shrinks, rotates in one piece as if it were solid, and remains incompressible and homogeneous. The condition is of course artificial, but it represents the actual processes of nature as well as may be, consistently with the postulated incompressibility and homogeneity[1].

The shrinkage of a constant mass of matter involves an increase of its density, and we have therefore to trace the changes which supervene as the star shrinks, and as the liquid of which it is composed increases in density. The shrinkage will, in ordinary parlance, bring the weights nearer to the axis of rotation. Hence in order to keep up the rotational momentum, which as we have seen must remain constant, the mass must rotate quicker. The greater speed of rotation augments the importance of centrifugal force compared with that of gravity, and as the flattening of the planetary spheroid was due to centrifugal force, that flattening is increased; in other words the ellipticity of the planetary spheroid increases.

As the shrinkage and corresponding increase of density proceed, the planetary spheroid becomes more and more elliptic, and the succession of forms constitutes a family classified according to the density of the liquid. The specific mark of this family is the flattening or ellipticity.

Now consider the stability of the system. We have seen that the spheroid with a slow rotation, which forms our starting-point, was slightly less stable than the sphere, and as we proceed through the family of ever flatter ellipsoids the stability continues to diminish. At length when it has assumed the shape shown in Fig. 2, where the equatorial and polar axes are proportional to the numbers 1000 and 583, the stability has just disappeared. According to the general principle explained above this is a form of bifurcation, and corresponds to the form denoted A. The specific difference a of this family must be regarded as the excess of the ellipticity of this figure above that of all the earlier ones, beginning with the slightly flattened planetary spheroid. Accordingly the specific difference a of the family has gradually diminished from the beginning and vanishes at this stage.

[1] Mathematicians are accustomed to regard the density as constant and the rotational momentum as increasing. But the way of looking at the matter, which I have adopted, is easier of comprehension, and it comes to the same in the end.

According to Poincaré's principle the vanishing of the stability serves us with notice that we have reached a figure of bifurcation, and it becomes necessary to inquire what is the nature of the specific difference of the new family of figures which must be coalescent with the old one at this stage. This difference is found to reside in the fact that the equator, which in the planetary family has hitherto been circular in section, tends to become elliptic. Hitherto the rotational momentum has been kept up to its constant value partly by greater speed of rotation and partly by a symmetrical bulging of the equator. But now while the speed of rotation still increases[1], the equator tends to bulge outwards at two diametrically opposite points and to be flattened midway between these protuberances. The specific difference in the new family, denoted in the general

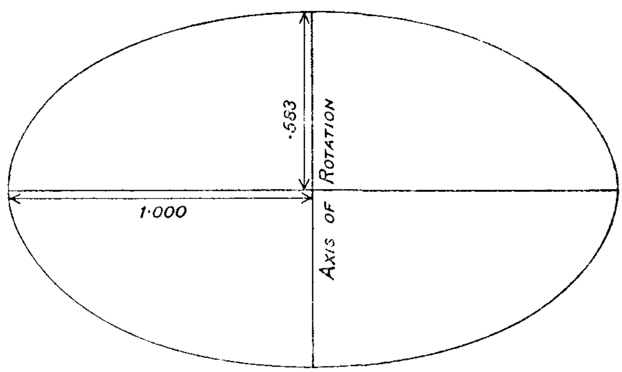

Fig. 2.

Planetary spheroid just becoming unstable.

sketch by b, is this ellipticity of the equator. If we had traced the planetary figures with circular equators beyond this stage A, we should have found them to have become unstable, and the stability has been shunted off along the $A+b$ family of forms with elliptic equators.

This new series of figures, generally named after the great mathematician Jacobi, is at first only just stable, but as the density increases the stability increases, reaches a maximum and then declines. As this goes on the equator of these Jacobian figures becomes more and more elliptic, so that the shape is considerably elongated in a direction at right angles to the axis of rotation.

[1] The mathematician familiar with Jacobi's ellipsoid will find that this is correct, although in the usual mode of exposition, alluded to above in a footnote, the speed diminishes.

At length when the longest axis of the three has become about three times as long as the shortest[1], the stability of this family of figures vanishes, and we have reached a new form of bifurcation and must look for a new type of figure along which the stable development will presumably extend. Two sections of this critical Jacobian figure, which is a figure of bifurcation, are shown by the dotted lines in Fig. 3; the upper figure is the equatorial section at right angles to the axis of rotation, the lower figure is a section through the axis.

Now Poincaré has proved that the new type of figure is to be derived from the figure of bifurcation by causing one of the ends to be prolonged into a snout and by bluntening the other end. The

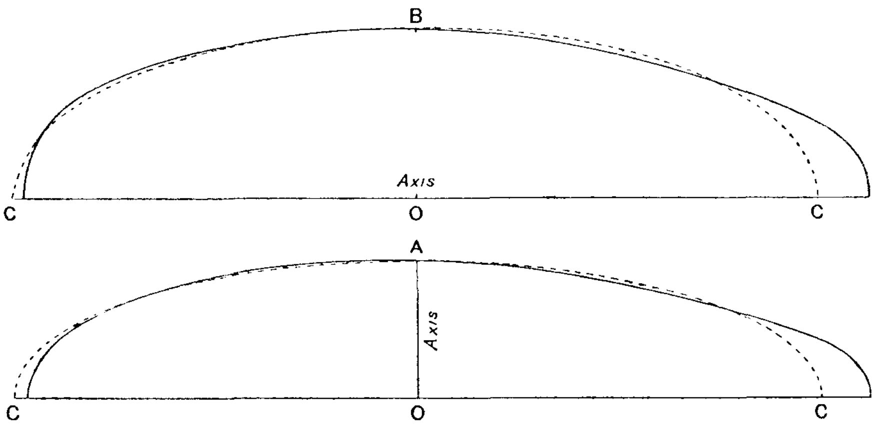

Fig. 3.

The "pear-shaped figure" and the Jacobian figure from which it is derived.

snout forms a sort of stalk, and between the stalk and the axis of rotation the surface is somewhat flattened. These are the characteristics of a pear, and the figure has therefore been called the "pear-shaped figure of equilibrium." The firm line in Fig. 3 shows this new type of figure, whilst, as already explained, the dotted line shows the form of bifurcation from which it is derived. The specific mark of this new family is the protrusion of the stalk together with the other corresponding smaller differences. If we denote this difference by c, while $A+b$ denotes the Jacobian figure of bifurcation from which it is derived, the new family may be called $A+b+c$, and c is zero initially. According to my calculations this series of figures is stable[2],

[1] The three axes of the ellipsoid are then proportional to 1000, 432, 343.

[2] M. Liapounoff contends that for constant density the new series of figures, which M. Poincaré discovered, has less rotational momentum than that of the figure of bifurcation. If he is correct, the figure of bifurcation is a limit of stable figures, and none can

but I do not know at what stage of its development it becomes unstable.

Professor Jeans has solved a problem which is of interest as throwing light on the future development of the pear-shaped figure, although it is of a still more ideal character than the one which has been discussed. He imagines an *infinitely* long circular cylinder of liquid to be in rotation about its central axis. The existence is virtually postulated of a demon who is always occupied in keeping the axis of the cylinder straight, so that Jeans has only to concern himself with the stability of the form of the section of the cylinder, which as I have said is a circle with the axis of rotation at the centre. He then supposes the liquid forming the cylinder to shrink in diameter, just as we have done, and finds that the speed of rotation must increase so as to keep up the constancy of the rotational momentum. The circularity of section is at first stable, but as the shrinkage proceeds the stability diminishes and at length vanishes. This stage in the process is a form of bifurcation, and the stability passes over to a new series consisting of cylinders which are elliptic in section. The circular cylinders are exactly analogous with our planetary spheroids, and the elliptic ones with the Jacobian ellipsoids.

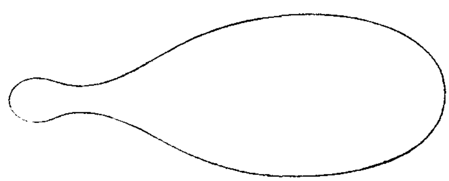

Fig. 4.
Section of a rotating cylinder of liquid.

With further shrinkage the elliptic cylinders become unstable, a new form of bifurcation is reached, and the stability passes over to a series of cylinders whose section is pear-shaped. Thus far the analogy is complete between our problem and Jeans's, and in consequence of the greater simplicity of the conditions, he is able to carry his investigation further. He finds that the stalk end of the pear-like section continues to protrude more and more, and the flattening between it and the axis of rotation becomes a constriction. Finally the neck breaks and a satellite cylinder is born. Jeans's figure for an advanced stage of development is shown in Fig. 4, but

exist with stability for greater rotational momentum. My own work seems to indicate that the opposite is true, and, notwithstanding M. Liapounoff's deservedly great authority, I venture to state the conclusions in accordance with my own work.

his calculations do not enable him actually to draw the state of affairs after the rupture of the neck.

There are certain difficulties in admitting the exact parallelism between this problem and ours, and thus the final development of our pear-shaped figure and the end of its stability in a form of bifurcation remain hidden from our view, but the successive changes as far as they have been definitely traced are very suggestive in the study of stellar evolution.

Attempts have been made to attack this problem from the other end. If we begin with a liquid satellite revolving about a liquid planet and proceed backwards in time, we must make the two masses expand so that their density will be diminished. Various figures have been drawn exhibiting the shapes of two masses until their surfaces approach close to one another and even until they just coalesce, but the discussion of their stability is not easy. At present it would seem to be impossible to reach coalescence by any series of stable transformations, and if this is so Professor Jeans's investigation has ceased to be truly analogous to our problem at some undetermined stage. However this may be this line of research throws an instructive light on what we may expect to find in the evolution of real stellar systems.

In the second part of this paper I shall point out the bearing which this investigation of the evolution of an ideal liquid star may have on the genesis of double stars.

II.

There are in the heavens many stars which shine with a variable brilliancy. Amongst these there is a class which exhibits special peculiarities; the members of this class are generally known as Algol Variables, because the variability of the star β Persei or Algol was the first of such cases to attract the attention of astronomers, and because it is perhaps still the most remarkable of the whole class. But the circumstances which led to this discovery were so extraordinary that it seems worth while to pause a moment before entering on the subject.

John Goodricke, a deaf-mute, was born in 1764; he was grandson and heir of Sir John Goodricke of Ribston Hall, Yorkshire. In November 1782, he noted that the brilliancy of Algol waxed and waned[1], and devoted himself to observing it on every fine night from the 28th December 1782 to the 12th May 1783. He communicated

[1] It is said that Georg Palitzch, a farmer of Prohlis near Dresden, had about 1758 already noted the variability of Algol with the naked eye. *Journ. Brit. Astron. Assoc.* Vol. xv. (1904—5), p. 203.

his observations to the Royal Society, and suggested that the variation in brilliancy was due to periodic eclipses by a dark companion star, a theory now universally accepted as correct. The Royal Society recognised the importance of the discovery by awarding to Goodricke, then only 19 years of age, their highest honour, the Copley medal. His later observations of β Lyrae and of δ Cephei were almost as remarkable as those of Algol, but unfortunately a career of such extraordinary promise was cut short by death, only a fortnight after his election to the Royal Society[1].

It was not until 1889 that Goodricke's theory was verified, when it was proved by Vogel that the star was moving in an orbit, and in such a manner that it was only possible to explain the rise and fall in the luminosity by the partial eclipse of a bright star by a dark companion.

The whole mass of the system of Algol is found to be half as great again as that of our sun, yet the two bodies complete their orbit in the short period of $2^d\ 20^h\ 48^m\ 55^s$. The light remains constant during each period, except for $9^h\ 20^m$ when it exhibits a considerable fall in brightness[2]; the curve which represents the variation in the light is shown in Fig. 7 below.

The spectroscope has enabled astronomers to prove that many stars, although apparently single, really consist of two stars circling around one another[3]; they are known as spectroscopic binaries. Campbell of the Lick Observatory believes that about one star in six is a binary[4]; thus there must be many thousand such stars within the reach of our spectroscopes.

The orientation of the planes of the orbits of binary stars appears to be quite arbitrary, and in general the star does not vary in brightness. Amongst all such orbits there must be some whose planes pass nearly through the sun, and in these cases the eclipse of one of the stars by the other becomes inevitable, and in each circuit there will occur two eclipses of unequal intensities.

It is easy to see that in the majority of such cases the two components must move very close to one another.

[1] *Dict. of National Biography*; article Goodricke (John). The article is by Miss Agnes Clerke. It is strange that she did not then seem to be aware that he was a deaf-mute, but she notes the fact in her *Problems of Astrophysics*, p. 337, London, 1903.

[2] Clerke, *Problems of Astrophysics*, p. 302 and ch. XVIII.

[3] If a source of light is approaching with a great velocity the waves of light are crowded together, and conversely they are spaced out when the source is receding. Thus motion in the line of sight virtually produces an infinitesimal change of colour. The position of certain dark lines in the spectrum affords an exceedingly accurate measurement of colour. Thus displacements of these spectral lines enables us to measure the velocity of the source of light towards or away from the observer.

[4] *Astrophysical Journ.* Vol. XIII. p. 89, 1901. See also A. Roberts, *Nature*, Sept. 12, 1901, p. 468.

The coincidence between the spectroscopic and the photometric evidence permits us to feel complete confidence in the theory of eclipses. When then we find a star with a light-curve of perfect regularity and with the characteristics of that of Algol, we are justified in extending the theory of eclipses to it, although it may be too faint to permit of adequate spectroscopic examination. This extension of the theory secures a considerable multiplication of the examples available for observation, and some 30 have already been discovered.

Dr Alexander Roberts, of Lovedale in Cape Colony, truly remarks that the study of Algol variables "brings us to the very threshold of the question of stellar evolution[1]." It is on this account that I propose to explain in some detail the conclusion to which he and some other observers have been led.

Although these variable stars are mere points of light, it has been proved by means of the spectroscope that the law of gravitation holds good in the remotest regions of stellar space, and further it seems now to have become possible even to examine the shapes of stars by indirect methods, and thus to begin the study of their evolution. The chain of reasoning which I shall explain must of necessity be open to criticism, yet the explanation of the facts by the theory is so perfect that it is not easy to resist the conviction that we are travelling along the path of truth.

The brightness of a star is specified by what is called its "magnitude." The average brightness of all the stars which can just be seen with the naked eye defines the sixth magnitude. A star which only gives two-fifths as much light is said to be of the seventh magnitude; while one which gives $2\frac{1}{2}$ times as much light is of the fifth magnitude, and successive multiplications or divisions by $2\frac{1}{2}$ define the lower or higher magnitudes. Negative magnitudes have clearly to be contemplated; thus Sirius is of magnitude $-1{\cdot}4$, and the sun is of magnitude -26.

The definition of magnitude is also extended to fractions; for example, the lights given by two candles which are placed at 100 ft. and 100 ft. 6 in. from the observer differ in brightness by one-hundredth of a magnitude.

A great deal of thought has been devoted to the measurement of the brightness of stars, but I will only describe one of the methods used, that of the great astronomer Argelander. In the neighbourhood of the star under observation some half dozen standard stars are selected of known invariable magnitudes, some being brighter and some fainter than the star to be measured; so that these stars afford a visible scale of brightness. Suppose we number them in order of increasing brightness from 1 to 6; then the observer estimates that on a given night his star falls between stars 2 and 3, on the next night, say between

[1] *Proc. Roy. Soc. Edinburgh*, XXIV. Pt. II. (1902), p. 73.

3 and 4, and then again perhaps it may return to between 2 and 3, and so forth. With practice he learns to evaluate the brightness down to small fractions of a magnitude, even a hundredth part of a magnitude is not quite negligible.

For example, in observing the star RR Centauri five stars were in general used for comparison by Dr Roberts, and in course of three months he secured thereby 300 complete observations. When the period of the cycle had been ascertained exactly, these 300 values were reduced to mean values which appertained to certain mean places in the cycle, and a mean light-curve was obtained in this way. Examples of light curves will be found in Figs. 5 and 7 below.

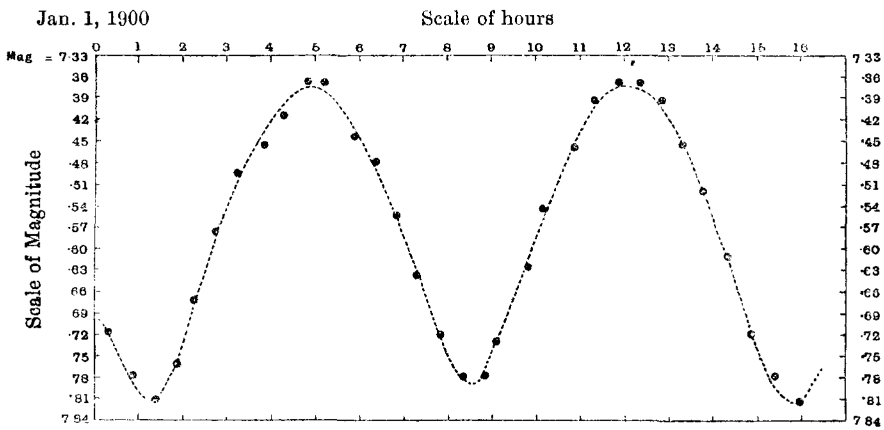

Fig. 5.

Light curve of RR Centauri.

I shall now follow out the results of the observation of RR Centauri not only because it affords the easiest way of explaining these investigations, but also because it is one of the stars which furnishes the most striking results in connection with the object of this essay[1]. This star has a mean magnitude of about $7\frac{1}{2}$, and it is therefore invisible to the naked eye. Its period of variability is $14^h\ 32^m\ 10^s{\cdot}76$, the last refinement of precision being of course only attained in the final stages of reduction. Twenty-nine mean values of the magnitude were determined, and they were nearly equally spaced over the whole cycle of changes. The black dots in Fig. 5 exhibit the mean values determined by Dr Roberts. The last three dots on the extreme right are merely the same as the first three on the extreme left, and are repeated to show how the next cycle would begin. The

[1] See *Monthly Notices R.A.S.* Vol. 63, 1903, p. 527.

smooth dotted curve will be explained hereafter, but, by reference to the scale of magnitudes on the margins of the figure, it may be used to note that the dots might be brought into a perfectly smooth curve by shifting some few of the dots by about a hundredth of a magnitude.

This light-curve presents those characteristics which are due to successive eclipses, but the exact form of the curve must depend on the nature of the two mutually eclipsing stars. If we are to interpret the curve with all possible completeness, it is necessary to make certain assumptions as to the stars. It is assumed then that the stars are equally bright all over their disks, and secondly that they are not surrounded by an extensive absorptive atmosphere. This last appears to me to be the most dangerous assumption involved in the whole theory.

Making these assumptions, however, it is found that if each of the eclipsing stars were spherical it would not be possible to generate

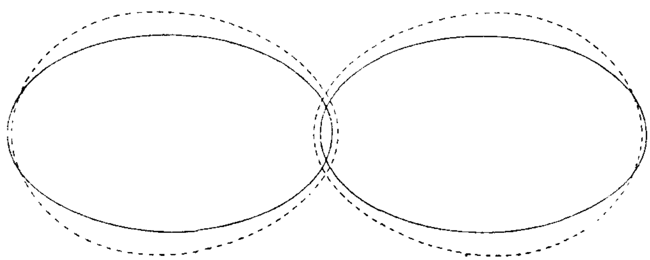

Fig. 6.
The shape of the star RR Centauri.

such a curve with the closest accuracy. The two stars are certainly close together, and it is obvious that in such a case the tidal forces exercised by each on the other must be such as to elongate the figure of each towards the other. Accordingly it is reasonable to adopt the hypothesis that the system consists of a pair of elongated ellipsoids, with their longest axes pointed towards one another. No supposition is adopted *à priori* as to the ratio of the two masses, or as to their relative size or brightness, and the orbit may have any degree of eccentricity. These last are all to be determined from the nature of the light-curve.

In the case of RR Centauri, however, Dr Roberts finds the conditions are best satisfied by supposing the orbit to be circular, and the sizes and masses of the components to be equal, while their luminosities are to one another in the ratio of 4 to 3. As to their shapes he finds them to be so much elongated that they overlap, as exhibited in his figure now reproduced as Fig. 6. The dotted curve shows a form of equilibrium of rotating liquid as computed by me some years before, and it was added for the sake of comparison.

On turning back to Fig. 5 the reader will see in the smooth dotted curve the light variation which would be exhibited by such a binary system as this. The curve is the result of computation and it is impossible not to be struck by the closeness of the coincidence with the series of black dots which denote the observations.

It is virtually certain that RR Centauri is a case of an eclipsing binary system, and that the two stars are close together. It is not of course proved that the figures of the stars are ellipsoids, but gravitation must deform them into a pair of elongated bodies, and, on the assumptions that they are not enveloped in an absorptive atmosphere and that they are ellipsoidal, their shapes must be as shown in the figure.

This light-curve gives an excellent illustration of what we have reason to believe to be a stage in the evolution of stars, when a single star is proceeding to separate into a binary one.

As the star is faint, there is as yet no direct spectroscopic evidence of orbital motion. Let us turn therefore to the case of another star, namely V Puppis, in which such evidence does already exist. I give an account of it, because it presents a peculiarly interesting confirmation of the correctness of the theory.

In 1895 Pickering announced in the *Harvard Circular* No. 14 that the spectroscopic observations at Arequipa proved V Puppis to be a double star with a period of $3^d\,2^h\,46^m$. Now when Roberts discussed its light-curve he found that the period was $1^d\,10^h\,54^m\,27^s$, and on account of this serious discrepancy he effected the reduction only on the simple assumption that the two stars were spherical, and thus obtained a fairly good representation of the light-curve. It appeared that the orbit was circular and that the two spheres were not quite in contact. Obviously if the stars had been assumed to be ellipsoids they would have been found to overlap, as was the case for RR Centauri[1]. The matter rested thus for some months until the spectroscopic evidence was re-examined by Miss Cannon on behalf of Professor Pickering, and we find in the notes on p. 177 of Vol. XXVIII. of the *Annals of the Harvard Observatory* the following: "A.G.C. 10534. This star, which is the Algol variable, V Puppis, has been found to be a spectroscopic binary. The period $1^d{\cdot}454$ (i.e. $1^d\,10^h\,54^m$) satisfies the observations of the changes in light, and of the varying separation of the lines of the spectrum. The spectrum has been examined on 61 plates, on 23 of which the lines are double." Thus we have valuable evidence in confirmation of the correctness of the conclusions drawn from the

[1] *Astrophysical Journ.* Vol. XIII. (1901), p. 177.

light-curve. In the circumstances, however, I have not thought it worth while to reproduce Dr Roberts's provisional figure.

I now turn to the conclusions drawn a few years previously by another observer, where we shall find the component stars not quite in contact. This is the star β Lyrae which was observed by Goodricke,

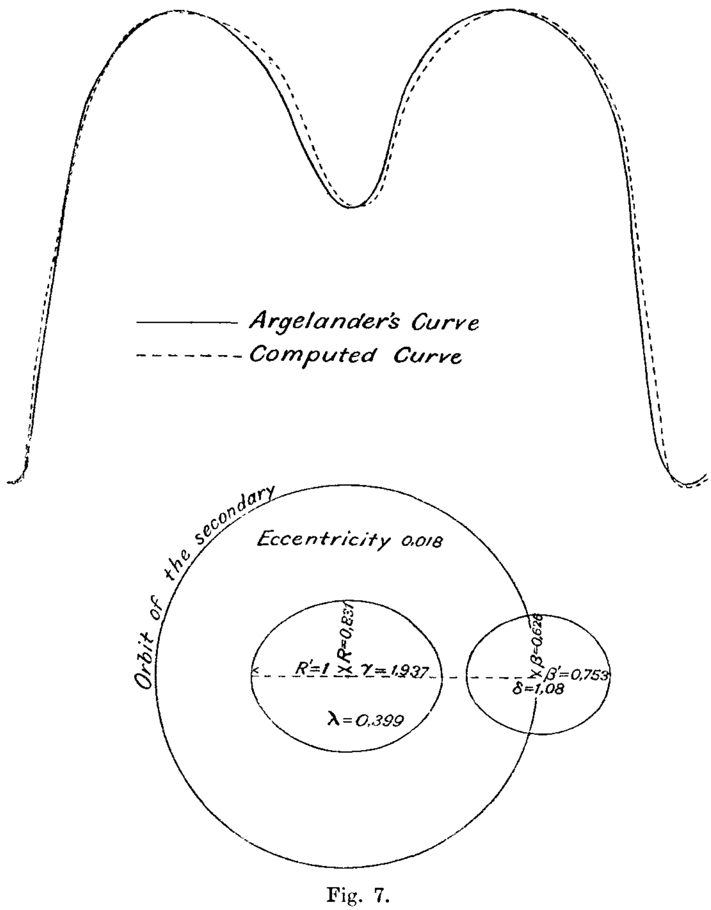

Fig. 7.
The light-curve and system of β Lyrae.

Argelander, Belopolsky, Schur, Markwick and by many others. The spectroscopic method has been successfully applied in this case, and the component stars are proved to move in an orbit about one another. In 1897, Mr G. W. Myers applied the theory of eclipses to the light-curve, on the hypothesis that the stars are elongated ellipsoids, and he obtained the interesting results exhibited in Fig. 7[1].

[1] *Astrophysical Journ.* Vol. VII. (1898), p. 1.

The period of β Lyrae is relatively long, being $12^d\,21^h\,47^m$, the orbit is sensibly eccentric, and the two spheroids are not so much elongated as was the case with RR Centauri. The mass of the system is enormous, one of the two stars being 10 times and the other 21 times as heavy as our sun.

Further illustrations of this subject might be given, but enough has been said to explain the nature of the conclusions which have been drawn from this class of observation.

In my account of these remarkable systems the consideration of one very important conclusion has been purposely deferred. Since the light-curve is explicable by eclipses, it follows that the sizes of the two stars are determinable relatively to the distance between them. The period of their orbital motion is known, being identical with the complete period of the variability of their light, and an easy application of Kepler's law of periodic times enables us to compute the sum of the masses of the two stars divided by the cube of the distance between their centres. Now the sizes of the bodies being known, the mean density of the whole system may be calculated. In every case that density has been found to be much less than the sun's, and indeed the average of a number of mean densities which have been determined only amounts to one-eighth of that of the sun. In some cases the density is extremely small, and in no case is it quite so great as half the solar density.

It would be absurd to suppose that these stars can be uniform in density throughout, and from all that is known of celestial bodies it is probable that they are gaseous in their external parts with great condensation towards their centres. This conclusion is confirmed by arguments drawn from the theory of rotating masses of liquid[1].

Although, as already explained, a good deal is known about the shapes and the stability of figures consisting of homogeneous incompressible liquid in rotation, yet comparatively little has hitherto been discovered about the equilibrium of rotating gaseous stars. The figures calculated for homogeneous liquid can obviously only be taken to afford a general indication of the kind of figure which we might expect to find in the stellar universe. Thus the dotted curve in Fig. 5, which exhibits one of the figures which I calculated, has some interest when placed alongside the figures of the stars in RR Centauri, as computed from the observations, but it must not be accepted as the calculated form of such a system. I have moreover proved more recently that such a figure of homogeneous liquid is unstable. Notwithstanding this instability it does not necessarily

[1] See J. H. Jeans, "On the density of Algol variables," *Astrophysical Journ.* Vol. XXII. (1905), p. 97.

follow that the analogous figure for compressible fluid is also unstable, as will be pointed out more fully hereafter.

Professor Jeans has discussed in a paper of great ability the difficult problems offered by the conditions of equilibrium and of stability of a spherical nebula[1]. In a later paper[2], in contrasting the conditions which must govern the fission of a star into two parts when the star is gaseous and compressible with the corresponding conditions in the case of incompressible liquid, he points out that for a gaseous star "the agency which effects the separation will no longer be rotation alone; gravitation also will tend towards separation....From numerical results obtained in the various papers of my own,...I have been led to the conclusion that a gravitational instability of the kind described must be regarded as the primary agent at work in the actual evolution of the universe, Laplace's rotation playing only the secondary part of separating the primary and satellite after the birth of the satellite."

It is desirable to add a word in explanation of the expression "gravitational instability" in this passage. It means that when the concentration of a gaseous nebula (without rotation) has proceeded to a certain stage, the arrangement in spherical layers of equal density becomes unstable, and a form of bifurcation has been reached. For further concentration concentric spherical layers become unstable, and the new stable form involves a concentration about two centres. The first sign of this change is that the spherical layers cease to be quite concentric and then the layers of equal density begin to assume a somewhat pear-shaped form analogous to that which we found to occur under rotation for an incompressible liquid. Accordingly it appears that while a sphere of liquid is stable a sphere of gas may become unstable. Thus the conditions of stability are different in these two simple cases, and it is likely that while certain forms of rotating liquid are unstable the analogous forms for gas may be stable. This furnishes a reason why it is worth while to consider the unstable forms of rotating liquid.

There can I think be little doubt but that Jeans is right in looking to gravitational instability as the primary cause of fission, but when we consider that a binary system, with a mass larger than the sun's, is found to rotate in a few hours, there seems reason to look to rotation as a contributory cause scarcely less important than the primary one.

With the present extent of our knowledge it is only possible to reconstruct the processes of the evolution of stars by means of

[1] *Phil. Trans. R.S.* Vol. CXCIX. A (1902), p. 1. See also A. Roberts, *S. African Assoc. Adv. Sci.* Vol. I. (1903), p. 6.

[2] *Astrophysical Journ.* Vol. XXII. (1905), p. 97.

inferences drawn from several sources. We have first to rely on the general principles of stability, according to which we are to look for a series of families of forms, each terminating in an unstable form, which itself becomes the starting-point of the next family of stable forms. Secondly we have as a guide the analogy of the successive changes in the evolution of ideal liquid stars; and thirdly we already possess some slender knowledge as to the equilibrium of gaseous stars.

From these data it is possible to build up in outline the probable history of binary stars. Originally the star must have been single, it must have been widely diffused, and must have been endowed with a slow rotation. In this condition the strata of equal density must have been of the planetary form. As it cooled and contracted the symmetry round the axis of rotation must have become unstable, through the effects of gravitation, assisted perhaps by the increasing speed of rotation[1]. The strata of equal density must then become somewhat pear-shaped, and afterwards like an hour-glass, with the constriction more pronounced in the internal than in the external strata. The constrictions of the successive strata then begin to rupture from the inside progressively outwards, and when at length all are ruptured we have the twin stars portrayed by Roberts and by others.

As we have seen, the study of the forms of equilibrium of rotating liquid is almost complete, and Jeans has made a good beginning in the investigation of the equilibrium of gaseous stars, but much more remains to be discovered. The field for the mathematician is a wide one, and in proportion as the very arduous exploration of that field is attained so will our knowledge of the processes of cosmical evolution increase.

From the point of view of observation, improved methods in the use of the spectroscope and increase of accuracy in photometry will certainly lead to a great increase in our knowledge within the next few years. Probably the observational advance will be more rapid than that of theory, for we know how extraordinary has been the success attained within the last few years, and the theory is one of extreme difficulty; but the two ought to proceed together hand in hand. Human life is too short to permit us to watch the leisurely procedure of cosmical evolution, but the celestial museum contains so many exhibits that it may become possible, by the aid of theory, to piece together bit by bit the processes through which stars pass in the course of their evolution.

[1] I learn from Professor Jeans that he now (December 1908) believes that he can prove that some small amount of rotation is necessary to induce instability in the symmetrical arrangement.

In the sketch which I have endeavoured to give of this fascinating subject, I have led my reader to the very confines of our present knowledge. It is not much more than a quarter of a century since this class of observation has claimed the close attention of astronomers; something considerable has been discovered already and there seems scarcely a discernible limit to what will be known in this field a century from now. Some of the results which I have set forth may then be shown to be false, but it seems profoundly improbable that we are being led astray by a Will-of-the-Wisp.